(12)

國立東華大學招生考試試題

招生學年度	九十八	招生類別	碩士班
系所班別	全球運籌管理研究所		
科目	微積分		
注意事項	本考科可使用掌上型計算機		

Notes. The whole paper is of 100 points. For every question, show your reasoning and steps to avoid losing a significant number of points.

#1. It is known that $\lim_{x\to 0}\frac{\sin x}{x}=1$.

(a). (5 points) Find $\lim_{x\to\infty}\sqrt{x}\sin\frac{1}{\sqrt{x}}$.

(b). (5 points) Find $\lim_{x\to 0}\frac{5x+\sin 3x}{x-6\sin x}$.

(c). (5 points) Find $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$.

(d). (5 points) Find $\lim_{x\to 0}\left(x-\frac{1}{\sqrt{x}}\right)\sin(\sqrt{x})$.

#2. In the following, m is a fixed positive constant and n takes up integer values as it tends to infinity.

(a). (5 points) Find $\lim_{n\to\infty}\sqrt{n^2+3n}-n$.

(b). (10 points) Find $\lim_{n\to\infty}\frac{m^n n!}{n^n}$. (*Note.* From Stirling's formula, $n!\sim\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$.)

#3. Let $\{a_n\}$ be a real-valued sequence such that $a_1=\sqrt{3}$ and $a_{n+1}=\sqrt{3+a_n}$ for $n\geq 1$.

(a). (5 points) Show that a_n is increasing in n.

(b). (5 points) Show that a_n is bounded above for all n.

(c). (5 points) From (a) and (b), $\lim_{n\to\infty}a_n$ exists. Find its value.

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#4. Find $\frac{dy}{dx}$ from the following functions.

(a). (5 points) $e^{\sin x} + y \ln x = x^2 y$.

(b). (5 points) $y = \tan x^x$. (*Note*. $\frac{d \tan x}{dx} = \sec^2 x$.)

#5(a). (5 points) Write down the Taylor expansion of $\cos(x + \delta)$ around x for small δ.

(b). (5 points) Estimate $\cos(\frac{\pi}{3} + 0.01)$ by the first three terms of its Taylor expansion, given that $\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

#6. Find the indefinite integral of the following functions where you can ignore the constant term.

(a). (5 points) $\int e^x \sin x \, dx$.

(b). (5 points) $\int x/(x^2 - 3)^2 \, dx$.

(c). (5 points) $\int 1/[(x-3)(x+2)] dx$.

#7. Let $f(x) = x$, $g(x) = x^2$, S be the region between $f(x)$ and $g(x)$ in the first quadrant.

(a). (5 points) Find the area of S.

(b). (5 points) Find the circumference of S. You can show your procedure without evaluating the exact value.

(c). (5 points) Find the volume of the object created by rotating S around the x axis.