招生學年度	101	招生類別	碩士班
系所班別	運籌管理研究所碩士班(乙組)		
科目	作業研究		
注意事項	本考科可使用掌上型計算機		

There are 5 Questions in this examination paper. Points allocated to parts are shown there. Show your steps and argument for all parts.

Note that there is a Chinese version of Question 5 in the Appendix.

#1. Problem **P**:

$$\begin{aligned} \max \quad & 10x + 15y + 16z, \\ s.t. \quad & 6x + 8y + 9z \le 21. \end{aligned}$$

Apply the following constraints to Problem **P** independently. Find the values of the variables and objective function at maximum.

(a). (5 points) $x \ge 0, y \ge 0, z \ge 0.$

(b). (5 points) $x \ge 0, y \le 0, z \ge 0.$

(c). (5 points) $x \ge 0.$

#2(a). (15 points) Solve Problem **P** in Question #1 by dynamic programming if $x, y, z \in \{0, 1, 2, \ldots\}$.

(b). (5 points) Solve Problem **P** in Question #1 if $x, y, z \in \{1, 2, \ldots\}$.

#3. Solve the following minimization problems. Find the values of the variables and the objective function at minimum.

(a). (5 points) Minimize $[y - (x-2)^2]^2$.

(b). (5 points) Minimize $[y - (x-2)^2]^2$ subject to $x + y \ge 4$.

(c). (5 points) Minimize $[y - (x-2)^2]^2 + [z - (x-4)^2]^2$ subject to $5x + y + z \ge 20$.

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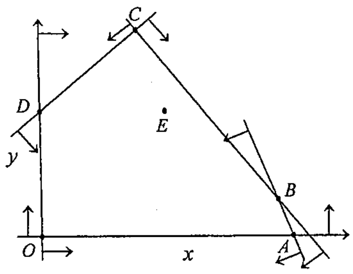
#4. The feasible set of a linear program is shown in the left-hand-side diagram, with constraints:

$$
\begin{aligned}
-x + y &\le 6, \\
x + y &\le 14, \\
2x + y &\le 26, \\
x, y &\ge 0.
\end{aligned}
$$



The (x, y) co-ordinates of vertices O, A, B, C, and D, the corner points of the feasible set, are (0, 0), (13, 0), (12, 2), (4, 10), and (0, 6), respectively. In addition, vertex E is an interior feasible point of co-ordinates (6, 6).

Let s_1, s_2, and s_3 be the slack variables of the first three constraints, and s_x and s_y be the surplus variables for the non-negativity constraints for the x-axle and y-axle, respectively. By definition,

$$
\begin{aligned}
-x + y + s_1 &= 6, \\
x + y + s_2 &= 14, \\
2x + y + s_3 &= 26, \\
x - s_x &= 0, \\
y - s_y &= 0.
\end{aligned}
$$

(a). (5 points) Find the values of $(x, y, s_1, s_2, s_3, s_x, s_y)$ for vertices O, C, and E.

(b). Suppose that the objective function is to maximize $x+py$, where p is a constant.

(i). (5 points) Find the range of values of p that makes vertex C a maximum point.

(ii). (5 points) Find the range of values of p that makes vertices B and C maximum points.

(c). Suppose that the objective function is to maximize $2y-x$.

(i). (5 points) What is the effect on the maximum value of the objective function if the third constraint is changed to $2x + y \le 25$?

(ii). (5 points) What is the effect on the maximum value of the objective function if the first constraint is changed to $-x + y \le 5$?

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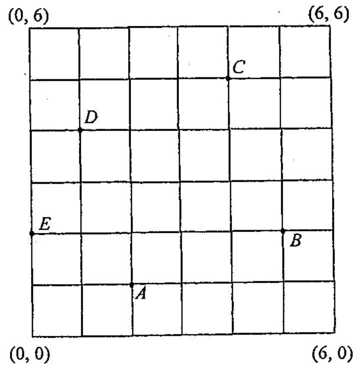
#5. The streets of a city run both east-west and north-south as shown in the left-hand-side diagram. Each segment of a street is of 1 km long. Consequently, the four corners of the city are labeled as co-ordinates (0, 0), (0, 6), (6, 0), and (6, 6), and the five retail outlets of a chain store at locations A to E of the city have co-ordinates (2, 1), (5, 2), (4, 5), (1, 4), and (0, 2), respectively.



To move within the city one needs to move along the street segments. As a result, the shortest distance between locations A and B is of 4 km; the shortest distance between locations A and E is of 3 km. Other distances can be found in the similar way.

The chain store is going to set up a depot to support the operations of the five retail outlets. Every day goods will be sent from the depot to the retail outlets. The objective is to find a location that minimizes the total distance from the depot to the retail outlets. It is possible to set up the depot at any location along the street segments, not necessarily at a junction of streets.

(a). First consider a simplified problem. Suppose that a depot is set up on or between locations B and E.

(i). (5 points) Find an optimal location of the depot to minimize its total distance from the depot to locations B and E.

(ii). (5 points) Now suppose that every day the number of visits from the depot to outlet at B is twice as much as to outlet at E. Find an optimal location of the depot to minimize its total traveling distance for serving the retail outlets at locations B and E.

(b). (5 points) Now suppose that every day there is one visit from the depot to each of the five retail outlets. From part (a) or otherwise, find an optimal location of the depot to minimize the total daily travel distance from the depot to the five outlets.

(c). (10 points) Irrespective of your answer to (b), assume that the depot is set up at location A. A truck is sent from location A to visit each of the other retail outlets once before returning to location A. Formulate an optimization problem to find a shortest-path route for the truck. *Solving the problem is not required.*

— THE END —

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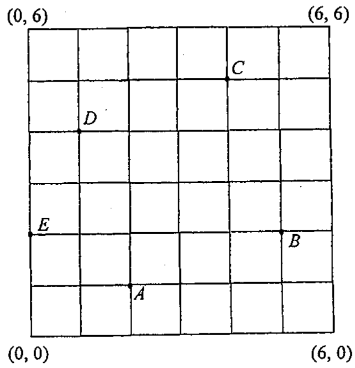
Appendix

以下是第五題的中文版本。

第五題：一個城市的街道東西和南北向如左圖。每一節街道都是 1 公里長。因此，城市的四角可標記爲(0, 0)、(0, 6)、(6, 0)與(6, 6)；另外，一連鎖店在城中點 *A* 到點 *E* 的五個零售點的坐標可分別記錄爲(2, 1)、(5, 2) 、(4, 5)、(1, 4)與(0, 2)。



在城中走動需要沿著街道的方向，所以，從點 *A* 到點 *B* 的最短距離是 4 公里；從點 *A* 到點 *E* 的最短距離是 3 公里；其他的距離也可以用類似的方式得到。

連鎖店將設立一個倉庫輔助 5 個零售點的業務，每天將貨品從倉庫送到各零售點。倉庫的選址，是以從倉庫到 5 個零售點的總距離最短爲目標，而倉庫可設於街道上任何地點，不一定要在街角街道相交的地方。

甲、首先考慮一個簡化的問題：假設倉庫設於點 *B* 和點 *E* 之間。

(i). (5 分) 找出倉庫的最佳位置，使得從倉庫到點 *B* 和點 *E* 的總距離最短。

(ii). (5 分) 假設每天從倉庫到零售點 *B* 的次數是從倉庫到零售點 *E* 次數的兩倍。找出倉庫的最佳位置，使得從倉庫運送貨品到零售點 *B* 和零售點 *E* 的總距離最短。

乙、(5 分) 假設每天都從倉庫送一次貨到每個零售點。利用甲部的結果或其他的方法，找出倉庫的最佳位置，使得每天從倉庫運送貨品到零售點的總運貨路程最短。

丙、(10 分) 不管乙部的答案爲何，假設倉庫是設於零售點 *A*：一輛貨車從零售點 *A* 出發，送貨到其他零售點，每點一次，然後回到零售點 *A*。請給出一個找尋貨車最短路路徑的優化模型。*不用解模型*。